Charge localization and dynamical spin locking in double quantum dots driven by ac magnetic fields

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In this work we investigate electron localization and dynamical spin locking induced by ac magnetic fields in double quantum dots. We demonstrate that by tuning the ac magnetic fields parameters, i.e., the field intensity, frequency and the phase difference between the fields within each dot, coherent destruction of tunneling (and thus charge localization) can be achieved. We show that in contrast with ac electric fields, ac magnetic fields are able to induce spin locking, i.e., to freeze the electronic spin, at certain field parameters. We show how the symmetry of the Hamiltonian determines the quasienergy spectrum which presents degeneracies at certain field parameters, and how it is reflected in the charge and spin dynamics.

PACS numbers:

Introduction: Quantum coherent effects in mesoscopic systems, such as quantum dots, are a subject of great current interest, both from the theoretical point of view, and because of a growing number of possible applications. Experimental successes in detecting Rabi oscillations driven by electric ac-fields in single quantum dots (QD's)[1] have spurred interest in the use of intense ac-fields to coherently manipulate the time development of electronic states[2]. An exciting possibility is to make use of the phenomenon of coherent destruction of tunneling (CDT)[3–9], in which the tunneling dynamics of a quantum system becomes suppressed at certain values of the intensity and frequency of the periodic electric field. CDT has been theoretically analyzed for ac electric field driven systems such as double quantum wells [4, 5, 10],superlattices[11], double quantum dots (DQD's)[12, 13], arrays of QD's[14, 15], nanoelectromechanical systems, as triple vibrating QD's[16], strongly correlated two dimensional QD's[17] and Bose-Einstein condensates[18, 19]. Also recent experiments on periodically driven cold atoms show super-Bloch oscillations which present a strong dependence on the phase of the potencial driving[20].

Spin qubits, consisting of two-level systems, can also be coherently manipulated in DQD's. Electron spin resonance experiments[21–23] measure coherent spin rotations of one single electron, a fundamental ingredient for quantum operations. In the present work we propose an additional way to control and manipulate spin qubits in double quantum dots driven by ac periodic magnetic fields, by tuning the intensity, frequency and relative phase of the ac magnetic fields within each quantum dot. As we will see below, under particular field configurations, charge localization within either the left or the right dot can be induced. Furthermore, we will show how the ac magnetic field can be tuned in order to lock the electron spin in its initial spin state.

In the present work we analyze the effect of crossed dc and ac magnetic fields on the electron dynamics in a double quantum dot. The static field induces a spin splitting within each dot and the ac magnetic field, which we consider linearly polarized, induces coherent spin rotations. We will show that by tuning the parameters of the ac magnetic fields: field intensity, frequency and phase, applied to the DQD, coherent destruction of tunneling (and therefore charge localization) can be achieved. We will show as well that ac magnetic fields allow, in contrast with ac electric fields, to lock the electron spin in the initial spin state at particular sets of ac field parameters. How the system can be tuned to reach independently one or the other, or both simultaneously will be discussed.
We consider Floquet theory[24] to analyze the electron dynamics in isolated DQD's, in the presence of crossed dc and ac magnetic fields. We show how the different symmetries of the Hamiltonian determine the conditions for inducing charge localization or dynamical spin locking, and how they are related with the quasienergy spectrum. Our results cover the case of different Zeeman splitting within each dot. This configuration arises for instance, for vertical dots grown with different materials (g-factor engineering[25]) or due to the inhomogeneous Overhauser field produced by Hyperfine interaction[26–28].

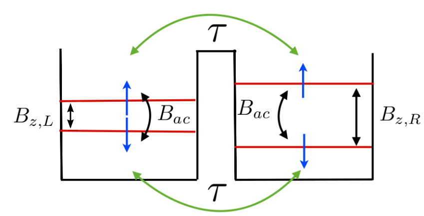

Figure 1: Double quantum dot with crossed ac and dc magnetic fields: $\mathbf{B}(t) = (B_{ac}(t), 0, B_z)$. The Zeeman splitting in the left and right dot is given by $B_{z,L}$ and $B_{z,R}$ respectively. The parameter τ describes the tunneling between the dots.

Model: The system we consider is a double quantum dot with one electron (Fig.1), subjected to crossed linearly polarized $B_{ac}(t)$, and a B_z magnetic fields.

The Hamiltonian reads:

$$H(t) = H_0 + H_\tau + H^B_{dc} + H^B_{ac}(t) \tag{1}$$

$$\begin{aligned} H_0 &= \sum_{i,\sigma} \epsilon_{g,i} c^\dagger_{\sigma,i} c_{\sigma,i} \\ H_\tau &= -\tau \sum_{i\neq j} \left(c^\dagger_{\sigma,i} c_{\sigma,j} + h.c. \right) \\ H^B_{ac}(t) &= \sum_i B_{ac,i} S_{x,i} \sin(\omega t + \phi_i) \\ H^B_{dc} &= \sum_i B_{z,i} S_{z,i} \end{aligned}$$

where the index $i = L, R$ (left/right), refers to the position in the system, $\epsilon_{g,i}$ is the gate voltage in the i dot, $\sigma = \uparrow, \downarrow$ is the spin projection in the basis of S_z eigenstates, τ is the interdot tunneling parameter, $\mathbf{S}_i = (S_{x,i}, S_{y,i}, S_{z,i})$ is the spin operator in second quantization at the i position, and the intensities of the ac and static magnetic fields within each dot are given by $B_{ac,i}$ and $B_{z,i}$ respectively. Also, in difference with ac electric fields, and due to the Hamiltonian symmetry in the present case, the introduction of the phase parameter in the magnetic field within each dot, ϕ_i, will allow the definition of the generalized parity symmetry (GP), i.e. a $\mathbb{Z}_2$ symmetry group different of parity symmetry.

Previous works have shown that the GP symmetry, defined as: $\{x \to -x,\ t \to t + T/2\}$, is important for CDT[5, 29, 30]. This parity operation defines a $\mathbb{Z}_2$ symmetry group, classifying the solutions as even or odd. Our aim in this paper is to introduce GP invariance in our system in order to localize and manipulate spin qubits.

The Floquet theorem asserts that a general solution for a system with T periodicity can be written as $|\Psi_\alpha(x,t)\rangle = e^{-i\varepsilon_\alpha t/\hbar}|\phi_\alpha(x,t)\rangle$, where ε_α is the quasienergy, and $|\phi_\alpha\rangle$ is the Floquet state with the property: $|\phi_\alpha(x,t)\rangle = |\phi_\alpha(x,t+T)\rangle$. Because our results are obtained by means of Floquet theory, we can consider GP symmetry as the natural extension of parity symmetry $\{x \to -x\}$, to the composed Hilbert space $H \otimes \mathcal{T}$ or Sambe space[31], where $\mathcal{T}$ is the Hilbert space of all T-periodic functions. The Floquet states are obtained by solving the Floquet Hamiltonian: $\mathcal{H}(t) = H(t) - i\hbar\partial_t$, and the scalar product is defined in Sambe space as: $\langle\langle \phi_\alpha(x,t) | \phi_\beta(x,t)\rangle\rangle = \frac{1}{T}\int_0^T \langle \phi_\alpha(x,t) | \phi_\beta(x,t)\rangle dt$.

Applying the GP operation to the Hamiltonian (Eq.1), we obtain invariance for: $B_{z,L} = B_{z,R}$, $B_{ac,L} = B_{ac,R}$, $\epsilon_{g,L} = \epsilon_{g,R}$ and $\phi = \phi_R - \phi_L = \pi$ (we consider $\epsilon_{g,L} = \epsilon_{g,R}$ from now on). We can also define a $\mathbb{Z}_2$ invariance within a single dot: writing the single dot Hamiltonian in the S_x eigenstates basis, it is clear that the system is invariant under $\{|\uparrow\rangle \leftrightarrow |\downarrow\rangle,\ t \to t + T/2\}$, this is an internal $\mathbb{Z}_2$ symmetry of the single dot Hamiltonian. We call this $\mathbb{Z}_2$ symmetry *generalized spin parity* (GSP).

If $\phi = 0$ the Hamiltonian is parity invariant, but a change of sign in all time dependent terms appears due to the addition of a semiperiod, while the time independent terms remain with the same sign, breaking the GP symmetry. Therefore, the difference of the phase parameter determines if the symmetry of the system corresponds to a parity invariant ($\phi = 0$) or a GP invariant ($\phi = \pi$) Hamiltonian. Approaches of quasienergies are frequently signatures of electron localization via the suppression of tunneling. We will see below that degeneracies of the quasienergies with opposite GP gives rise to CDT, and degeneracies with opposite GSP result in spin locking, i.e., if the electron is initially in some S_x eigenstate, it will remain locked in that particular spin projection.

Results: The present configuration is not analytically solvable. We employ numerical methods and perturbation theory in the Floquet formalism in the high frequency limit [5, 32], considering the Zeeman splitting and the tunneling as the perturbation (up to first order), both in the same footing. By high frequency limit we mean that $\omega \gg \tau, B_z$. We obtain analytically the quasienergies ε_α, and numerically the time evolution of the electronic states occupation.

Fig.2(top) shows the quasienergy spectrum with symmetric Zeeman splittings ($B_{z,L} = B_{z,R}$), and $\phi = \pi$ within the first Brillouin zone, in the high frequency limit. The parameters considered are of the order of those typical for transport experiments in QDs ([23]).
The quasienergies obtained by means of perturbation theory become:

$$\varepsilon_{s,s'} = \frac{1}{2}(\pm B_z \pm 2\tau) J_0\left(\frac{B_{ac}}{\omega}\right), \quad (s, s' = \pm) \tag{2}$$

The crossings of all the quasienergies in several points, according to the Wigner-Von Neumann theorem[33], are related to quasienergies that belong to different symmetry groups. This fact reflect the classification of the quasienergies in a $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ symmetry group (GSP and GP), where all the Floquet states are orthogonal to each other. CDT and also dynamical spin locking has been found in this configuration (Fig.2(bottom)) for values of the frequency and intensity of the field such that $J_0(B_{ac}/\omega) = 0$ (Eq.2). Crossings between quasienergies with opposite GP result in charge localization in the left or right dot, while crossings between quasienergies with opposite GSP result in dynamical spin locking for the eigenstates of the S_x matrix. If we consider instead, that $\phi = 0$, where the fields are in phase, the quasienergy spectrum changes, disappearing the multiple crossings (Fig.3).

The measurement of the occupation probabilities shows that CDT does not occur in this case, but dynamically induced spin locking is achieved. These results indicate that CDT is strongly dependent of the spatial inhomogeneity introduced by the difference of phase ϕ, while spin localization is not.

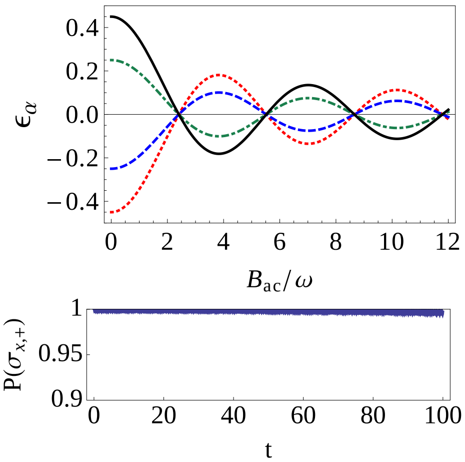

Figure 2: (top) Quasienergies versus B_{ac}/ω for $\phi=\pi$. Note the crossings of all the quasienergies due to the existence of two generalized parity symmetries $\mathbb{Z}_2$ (GP and GSP) for $J_0\left(B_{ac}/\omega\right)=0$. (bottom) Occupation probability versus time for the initial state $|\Psi^L_{x,+}\rangle=\left(|\uparrow\rangle_L+|\downarrow\rangle_L\right)/\sqrt{2}$, at the first crossing of the quasienergies ($B_{ac}/\omega\simeq 2.404$). The probability is almost constant in time, and the electron remains in a well defined spin projection and spatially localized in the left dot. In this case, both CDT (spatial localization) and dynamical spin locking occurs at the first zero of $J_0\left(B_{ac}/\omega\right)$. Parameters: $B_{z,L}=B_{z,R}=0.7$, $\omega=8$ and $\tau=0.1$ in units $\mu_B=\hbar=1$. These parameters correspond to a $\omega=16\mu$eV, $B_z=1.4\mu$eV and $\tau=0.2\mu$eV.

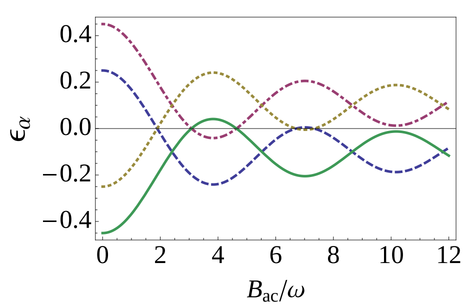

Figure 3: Quasienergies versus B_{ac}/ω for $\phi=0$. GP symmetry is broken. Crossings between quasienergies with opposite GSP appear, but the quasienergies with opposite parity, do not cross due to a 2τ splitting. Therefore, at the first zero of J_0, CDT does not happen, and just spin locking is achieved. $|\Psi^L_{x,+}\rangle$ is the initial state, being $|\Psi^{i=L,R}_{x,\pm}\rangle=\left(|\uparrow\rangle_i\pm|\downarrow\rangle_i\right)/\sqrt{2}$. In this case, the electron performs Rabi oscillations between $|\Psi^L_{x,+}\rangle$ and $|\Psi^R_{x,+}\rangle$, remaining in the initial spin projection. Parameters: $B_{z,L}=B_{z,R}=0.7$, $\omega=8$ and $\tau=1/10$.

Now we consider an spatial inhomogeneity in the Zeeman splittings (i.e. $B_{z,L}\neq B_{z,R}$), such that the parity and GP symmetries are broken, resulting in avoided crossings between quasienergies with opposite GP.

We have obtained, by means of perturbation theory, the quasienergies for arbitrary Zeeman splittings ($B_{z,L}\neq$

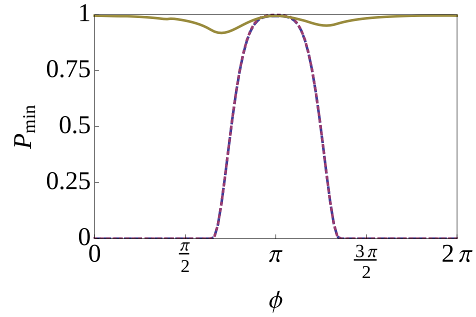

Figure 4: (Color online) P^k_{min} versus the difference of phase for the first zero of $J_0\left(B_{ac}/\omega\right)$, i.e. $B_{ac}/\omega\simeq 2.404$. Dashed line shows initial state freezing (P^1_{min}, red), dotted line shows the spatial localization (P^2_{min}, blue), while the continuous line shows the spin locking (P^3_{min}, brown). In this case P^1_{min} and P^2_{min} are coincident. As the phase ϕ moves away from π, $P^{k=1,2}_{min}$ decreases, reducing and even destroying CDT. Spin localization (P^3_{min}) holds for all ϕ (the state remains with a well defined spin projection, but oscillating within the dots), because, although GP symmetry is broken as ϕ varies from π, GSP remains. Parameters considered: $\omega=8$, $B_{z,L}=B_{z,R}=0.7$, and $\tau=1/10$.

$B_{z,R}$) and their dependence with ϕ:

$$\begin{aligned}\varepsilon_{i,j} &= \pm\frac{1}{2}\frac{B_{z,L}+B_{z,R}}{2}J_0\left(\frac{B_{ac}}{\omega}\right) \\ &\pm\frac{1}{2}\sqrt{\Delta_Z^2 J_0^2\left(\frac{B_{ac}}{\omega}\right)+\left(2\tau\right)^2 J_0^2\left(\frac{B_{ac}}{\omega}\sin\left(\phi/2\right)\right)}.\end{aligned} \quad (3)$$

The indices $i,j=\pm,\pm$ label the parity, according to each symmetry group (i refers to the GP and j refers to the GSP), and $\Delta_Z=\left(B_{z,L}-B_{z,R}\right)/2$. A very good agreement between the analytical expression (3) and the numerical calculation has been found.

In order to analyze the ϕ dependence at fixed intensity, we define the states $|\Psi^{L,R}_{x,\pm}\rangle=\left(|\uparrow\rangle_{L,R}\pm|\downarrow\rangle_{L,R}\right)/\sqrt{2}$ and the measure P^k_{min}, that takes the minimum value over one hundred periods to be in either the initial state $|\Psi^L_{x,+}\rangle$ ($k=1$), the left dot ($k=2$), or at the initial spin state projection, delocalized between the left and right dot ($k=3$), when the system time evolves from the initial state. Formally can be defined as $P^k_{min}=\min\left\{\sum_i\left|\langle\Psi^i_k|U\left(t,0\right)|\Psi^L_{x,+}\rangle\right|^2,\ t\in[0,100T]\right\}$, being $U\left(t,0\right)$ the time evolution operator obtained numerically from the Floquet states and i the number of states composing the subspace we are measuring, ie. $k=1\rightarrow i=1$ being $|\Psi^1_1\rangle=|\Psi^L_{x,+}\rangle$, $k=2\rightarrow i=1,2$ being $|\Psi^{1,2}_2\rangle=|\Psi^L_{x,\pm}\rangle$ and $k=3\rightarrow i=1,2$ being $|\Psi^{1,2}_3\rangle=|\Psi^{L,R}_{x,+}\rangle$. Fig.4 shows the measure P^k_{min} as a function of the phase difference between the ac magnetic fields within each dot.

In Eq.(3) the Bessel functions have different arguments due to the phase difference ϕ. Therefore their

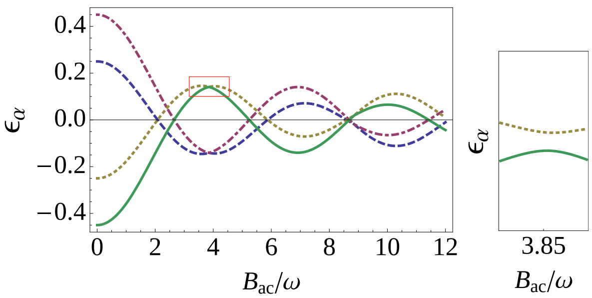

Figure 5: Quasienergies versus B_{ac}/ω for $\phi = 3\pi/7$. The avoided crossings arise from the lack of GP symmetry produced by the difference of phase $\phi \neq \pi$. Right figure shows a zoom of the framed area. Long time spatial localization is obtained in such avoided crossings, due to the nearby degeneracy. At $B_{ac} \simeq 5.5$ the crossing of quasienergies gives rise to dynamically induced spin locking. Also for $B_{ac} \simeq 8.8$ a nearby degeneracy between all four quasienergies occurs, giving rise to charge and spin locking. This is in good agreement with Eq.(4) for $n = 3$, where spatial and spin localization occur. $B_{z,L} = B_{z,R} = 0.7$, and $\tau = 1/10$.

zeros will be shifted, matching just at some intensities, where $J_0(B_{ac}/\omega) = J_0(B_{ac}\sin(\phi/2)/\omega) = 0$; i.e. exactly at intensities where spin locking and charge localization are recovered. In the high intensity limit (i.e. $B_{ac} \gg \omega \gg \tau, B_z$) we find these values to be:

$$\frac{B_{ac}}{\omega} = \frac{\pi(4n-1)}{4\sin(\phi/2)}, \ (n \in \mathbb{Z}, n > 0). \qquad (4)$$

Where we have used the asymptotic limit $J_0(x) \underset{x\gg1}{\sim} \sqrt{2/(\pi x)}\sin(x+\pi/4)$. The case $n = 3$ is shown in Fig.5 for $B_{ac}/\omega \simeq 8.8$.

In case of $\phi \neq \pi$ and $\phi \neq 0$, we can also consider avoided crossings between quasienergies with opposite GP parity out of the zeros of J_0 (inset in Fig.5, shows $\varepsilon_{+,\pm}$ and $\varepsilon_{-,\pm}$).

Fig.5 shows how just by tuning the intensity of the ac field one can switch between the different regimes, i.e. spatial localization regime ($B_{ac}/\omega \simeq 3.8$), dynamical spin locking ($B_{ac}/\omega \simeq 5.5$) and both ($B_{ac}/\omega \simeq 8.8$). The value of the P_k measured in the avoided crossing in Fig.5 for $B_{ac}/\omega \simeq 3.8$ are: $P^2_{min} = 0.98$ and $P^3_{min} = 0$, confirming just spatial charge localization and spin rotations.

Conclusions: We have shown that, in the high frequency regime, driving DQD's with ac magnetic fields allows to achieve independently charge localization and spin locking, by tuning the parameters of the ac magnetic fields. In particular, modifying the intensity or the phase difference of the ac magnetic fields applied to the quantum dots allows to select spatial localization, spin locking or both regimes simultaneously in the system, giving rise to a coherent control of the electronic states. In fact, the change of frequency can be used to modify the difference of phase of the ac magnetic fields between spatially separated dots.

We explain the different localization regimes in terms of the parity, GP or GSP symmetry in the system, which can be externally imposed and modified. Our results can be extended to other systems consisting on coupled two level systems with internal SU(2) and parity symmetry.

In summary, the combined localization effect of ac magnetic fields on the spin and spatial degrees of freedom allows a novel way for inducing, in a controlled way, charge localization and spin locking in double quantum dots. It opens a new road for qubits manipulation, a fundamental task for quantum information and quantum computation purposes. These results are also interesting for potential spintronic devices.

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